3 EDUCATIONAL MONOGRAPH

prepared from

Method for Estimating Ratio of Absorptance to Emittance 6

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FOREWORD

This Monograph was produced in a pilot program at Oklahoma State University in Stillwater, Oklahoma, under contract to the NASA Office of Technology Utilization. The program was organized to determine the feasibility of presenting the results of recent research in NASA Laboratories and under NASA contract in an educational format suitable as supplementary material in classwork at engineering colleges. The Monograph may result from editing single technical reports or synthesizing several technical reports resulting from NASA's research efforts.

Following the preparation of the Monographs, the program includes their evaluation as educational material in a number of universities throughout the country. The results of these individual evaluations in the classroom situation will be used to help determine if this procedure is a satisfactory way of speeding research results into engineering education.

ABSTRACT

A graphical method is presented for estimating the values of the ratio of absorptance to emittance α/ϵ that can be achieved with surfaces having a high degree of spectral selectivity. The ratio of emitting source to absorbing surface temperature is the parameter in the graphs. In principle, the results of the calculations presented are general and apply for any source or surface temperature. In practice, the ratios of absorptance to emittance so estimated can be used in radiant heat transfer calculations involving space vehicles. In this case α becomes α_s, the total normal absorptance of a surface to solar radiation, and ϵ the total hemispherical emittance.

Instructors Guide for Monographs

1. Education level of the Monograph--Undergraduate heat transfer course with section on radiant heat transfer or general heat transfer course in graduate level.

2. Prerequisite Course Material--Students should be at the level of calculation of radiant heat exchange.

3. Estimated number of lecture periods required--One hour of lecture.

4. Technical Significance--The material presented introduced the concept of spectrally selective surfaces for spacecraft temperature control or utilization of solar energy.

5. New concepts or unusual concepts--None.

6. How Monograph can best be used--

   (a) It is suggested that approximately a one hour lecture be given over the Monograph material.

   (b) It is suggested that the class be assigned home problems from the group listed.

7. Other literature of interest--Schmidt, R.N.; and Janssen, J.E.: Selective Coatings for Vacuum Stable High Temperature Solar Absorbers. Paper Presented at Symposium on Thermal Radiation of Solids, ASD, USAF, NBS, NASA, San Francisco (Calif.), Mar. 4-6, 1964.

8. Other reports reviewed by the editor in preparing this Monograph--Schmidt, R.N.; and Janssen, J.E.: Selective Coatings for Vacuum Stable High Temperature Solar Absorbers. Paper Presented at Symposium on Thermal Radiation of Solids, ASD, USAF, NBS, NASA, San Francisco (Calif.), Mar. 4-6, 1964.

9. Who to contact for further information--Technical Utilization Officer, Lewis Research Center, Cleveland, Ohio.

10. Note to instructor: All uncolored pages of the instructors Monograph are in the copies intended for student use.

INTRODUCTION

In environments where heat transfer to or from a surface is entirely or primarily through radiation, a knowledge of the surface absorptance α and the surface emittance ϵ is necessary for heat transfer calculations. When the internal heat fluxes to a surface are either zero or small compared with the external radiant heat fluxes, only the ratio of absorptance to emittance α/ϵ need be known to calculate the temperature or performance of a system. The principle example is a space vehicle exposed to sunlight. For example, the boiloff loss of cryogenic propellants from a well insulated tank in space is directly proportional to the ratio α_s/ϵ of the external surface of the tank [1]*.

Gray surfaces are those for which both the monochromatic absorptance α_λ and the monochromatic emittance ϵ_λ are invariant with wavelength, and therefore, for gray diffuse surfaces, $\alpha = \alpha_\lambda = \epsilon_\lambda = \epsilon$ and $\alpha/\epsilon = 1.0$. This is not the case for most materials. Values of α_s/ϵ for metals are greater than unity because their monochromatic absorptance and emittance decrease with increasing wavelength [2]. Therefore α_s is greater for the relatively short wavelength radiation from the sun than is ϵ for the longer wavelength radiation emitted from the much colder surface. Semi-conductors are similar to metals in that their α_s/ϵ is greater than one. Electrical nonconductors, have α_s/ϵ values less than unity because α_λ and ϵ_λ increase with increasing wavelength for this class of compounds.

Considerable research effort has been directed toward producing surfaces having ratios of α_s/ϵ either as high or as low as possible. High surface temperatures result from high values of α_s/ϵ and are most effective in increasing the performance of flat-plate collectors of solar energy [3]. Surfaces having low values of α_s/ϵ result in low surface temperatures and might be used, for example, on the previously mentioned cryogenic tanks.

ANALYSIS

An ideal spectrally selective surface is one for which the monochromatic absorptance α_λ (and the monochromatic emittance ϵ_λ) is unity over a span of wavelengths and changes abruptly to zero over the rest of the span. Figure 1 shows this behavior for the two types of ideal spectrally selective surfaces, types A and B. Type A surfaces with the high value for α_λ and ϵ_λ at short wave-

*Numbers in brackets denotes references

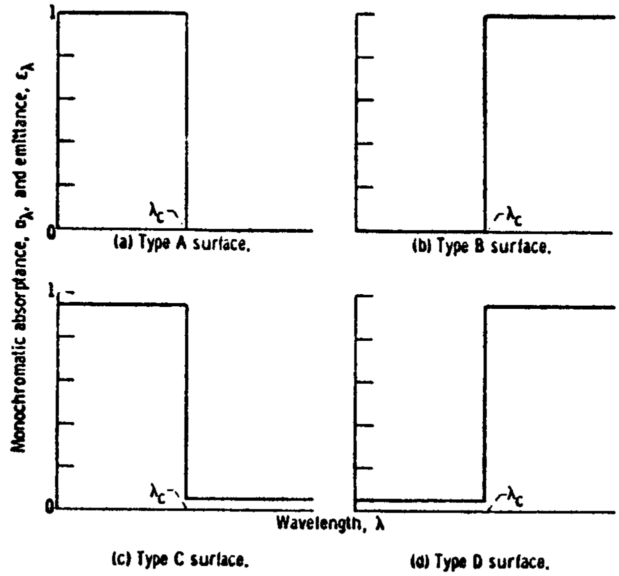

Figure 1. Absorptance and emittance of ideal spectrally selective surfaces.

lengths are absorbers for radiation from a high temperature emitter. Conversely, type B surfaces are reflectors for radiation from a high temperature emitter. These are sometimes characterized as solar absorbers or solar reflectors.

While type A and B surfaces are ideal in spectral selectivity, it is likely that more practical surfaces are those where a step change in α_λ is between 0.95 and 0.05. These are shown in Figure 1 as type C and D surfaces. Actual surfaces which approach the type C and D surfaces have been prepared. Examples of this are shown in Figure 2.

The spectrally selective surfaces shown in Figures 1 and 2 can be characterized by the wavelength at which the spectral absorptance or emittance changes abruptly from a high to a low value, or from a low to a high value. The wavelength of this step change in absorptance or emittance is termed λ_c.

The absorptance of a surface is a function of both the spectral selectivity of the surface and the source temperature of the spectral distribution of the incident flux. If the source is a black-body, the absorptance is given by

$$\alpha = \frac{\int_0^\infty \alpha_\lambda E_{b\lambda}(T_1)\, d\lambda}{\sigma T_1^4} \tag{1}$$

in which

$E_{b\lambda}(T_1)$ is the Planck's distribution of blackbody at temperature T_1

σ is the Stefan-Boltzmann constant.

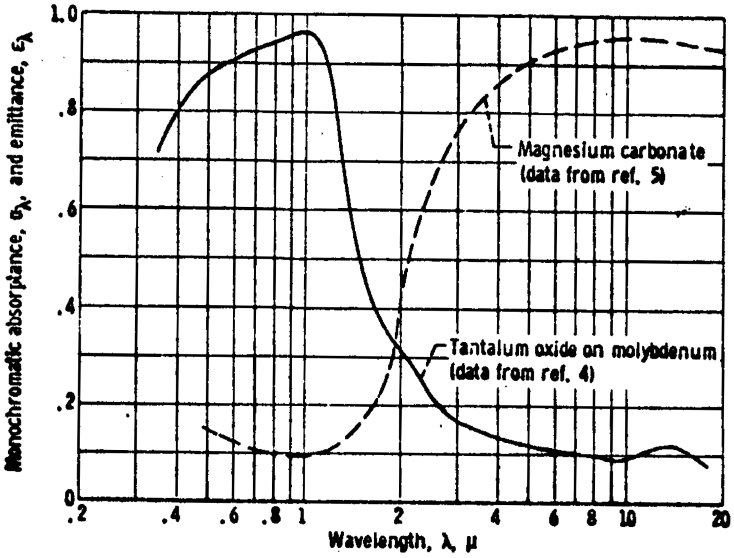

Figure 2. Absorptance and emittance of two spectrally selective surfaces.

Similarly, the emittance is given by

$$\epsilon = \frac{\int_o^\infty \epsilon_\lambda E_{b\lambda}(T_2)\ d\lambda}{\sigma T_1^4} \tag{2}$$

However, the fraction of the total energy radiated by a blackbody at all wavelengths shorter than a given value λ_o is a unique function of the product of wavelength and temperature $\lambda_o T$. This fraction is defined as

$$F_{\lambda,T} = \frac{\int_o^{\lambda_o} E_{b\lambda}(T)\ d\lambda}{\sigma T^4}$$

The fraction at wavelengths larger than λ_o is then

$$\frac{\int_{\lambda_o}^\infty E_{b\lambda}\ (T)\ d\lambda}{\sigma T^4} = 1-F_{\lambda,T}$$

If a surface has a high degree of spectral selectivity such that the monochromatic absorptance α_λ is substantially a constant value from $\lambda = 0$ to $\lambda_o = \lambda_c$ and a second constant value from $\lambda_o = \lambda_c$ to $\lambda = \lambda_\infty$, the α_λ term can be taken outside the integral in Equation 1. The absorptance can be calculated from

$$\alpha = \alpha_{o-\lambda_c} F_{\lambda_c,T} + \alpha_{\lambda_c-\infty}(1-F_{\lambda_c-T}) \tag{3}$$

by using values of $F_{\lambda,T}$ as a function of λT taken from tables [6]. Equation 2 can be similarly expressed such that the emittance can be calculated from

$$\epsilon = \epsilon_{o-\lambda_c} F_{\lambda_c,T} + \epsilon_{\lambda_c-\infty}(1-F_{\lambda_c,T}) \qquad (4)$$

The source temperature is used in evaluating the absorptance by Equation 3, and the surface temperature is used with Equation 4 for emittance.

Equations 3 and 4 have been used to calculate the absorptance or emittance of ideal spectrally selective surfaces for various values of $\lambda_c T$. These are plotted in Figure 3 for the four surfaces of Figure 1.

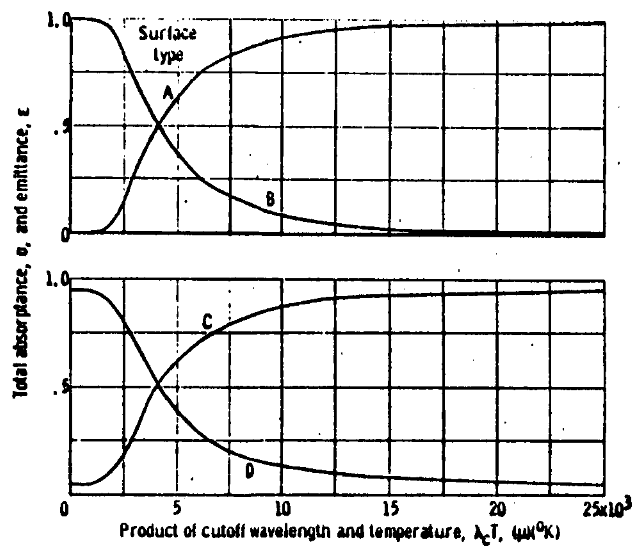

Figure 3. Total absorptance and emittance for ideal spectrally selective surfaces.

Figure 3 can be used to estimate α or ϵ for any surface where the monochromatic absorptance and emittance approach the step functions shown in Figure 1 and where the source and surface temperatures are known. For example, if a surface of type C with an abrupt change in α_λ or ϵ_λ at 0.85 microns is irradiated with sunlight (effective black body temperature 6000°K), the value of α from Figure 3 is 0.63. Similarly, if the same surface is at 500°K, the $\lambda_c T$ value is 425 micron-°K and the emittance from Figure 3 is 0.05. The α/ϵ ratio for this surface at these temperatures is 0.63/0.05 = 12.6.

The example just given indicates that it is also possible to plot α/ϵ as a function of $\lambda_c T$ by using temperature ratios as parameters. The example of α/ϵ = 12.6 for a temperature ratio T_1/T_2 of 6000/500 = 12.0 and a $\lambda_c T_1$ of 5100 micron-°K would be one point in developing the curves. Curves could also be based on a $\lambda_c T_2$ of 425 micron-°K in the preceding example by using the surface rather than the source temperature as the reference value.

Such curves were developed by using the source temperature T_1 in the $\lambda_c T$ term since most of the work in this area is related to sunlit systems where $6000^\circ K$ can be used at T_1. The alternative of using surface temperature would be less convenient since the surface temperature is often unknown and may be the quantity that is to be calculated after α/ϵ is established. The surface temperature T_2 must still be estimated to determine the temperature ratio parameter, but, for a sunlit surface operating below $500^\circ K$ ($T_1/T_2 > 12$), the results are relatively insensitive to changes in T_2.

Calculated α/ϵ values are presented in Figures 4(a) and 4(b) as a function of $\lambda_c T_1$ for ideal spectrally selective surfaces, that is, those with a step change in α_λ from unity to zero.

The results shown in Figures 4(a) and 4(b) are largely academic because surfaces of this degree of spectral selectivity are not presently attainable. A more realistic degree of spectral selectivity is seen in the type C and D surfaces, where α_λ and ϵ_λ change from 0.95 to 0.05. Calculated values for α/ϵ are presented in Figures 4(c) and 4(d) for these types of surfaces.

When the results for type A and C surfaces (Fig. 4(a) and 4(c)) are compared, it can be seen that curves of quite different shape are obtained. For type C surfaces α/ϵ does not increase continuously with decreasing $\lambda_c T$ but reaches a maximum value for each temperature ratio. At lower temperature ratios these maxima are quite sharp funtions of $\lambda_c T_1$ and show that there is an optimum value of λ_c for any given source temperature. Figure 4(c) can be used as a guide in developing a surface coating for any particular mission. At higher temperature ratios, the maximum α/ϵ is a less sensitive function of $\lambda_c T_1$.

The theoretical upper limit of α/ϵ that can be achieved with a type C surface is 19.0, and this only for very high temperature ratios and large values of $\lambda_c T_1$ beyond those shown in Figure 4(c). The practical upper limit is about 18, as shown for the range of $\lambda_c T_1$ and T_1/T_2 encompassed by this figure.

The type D surface (Fig. 5(d)) also gives curves quite differently shaped from those for the ideal type B surface of Figure 4(b). Minimum values of α/ϵ are shown as a function of $\lambda_c T_1$ and these minima are sharpest at low values of T_1/T_2. The lowest α/ϵ possible for a type D surface is 0.053, and this only for $\lambda_c T_1$ greater than that shown in Figure 4(d). A more reasonable lower limit is about 0.070 as shown in the figure.

Comparison with Present State of the Art

Figures 4(c) and 4(d) show the α/ϵ values that can be obtained with various ratios of source to surface temperature. These are based on a somewhat idealized description of surface spectral selectivity, and it is interesting to compare these predictions with what has been achieved experimentally. While no exhaustive literature search has been made, data are available from two recent reports that indi-

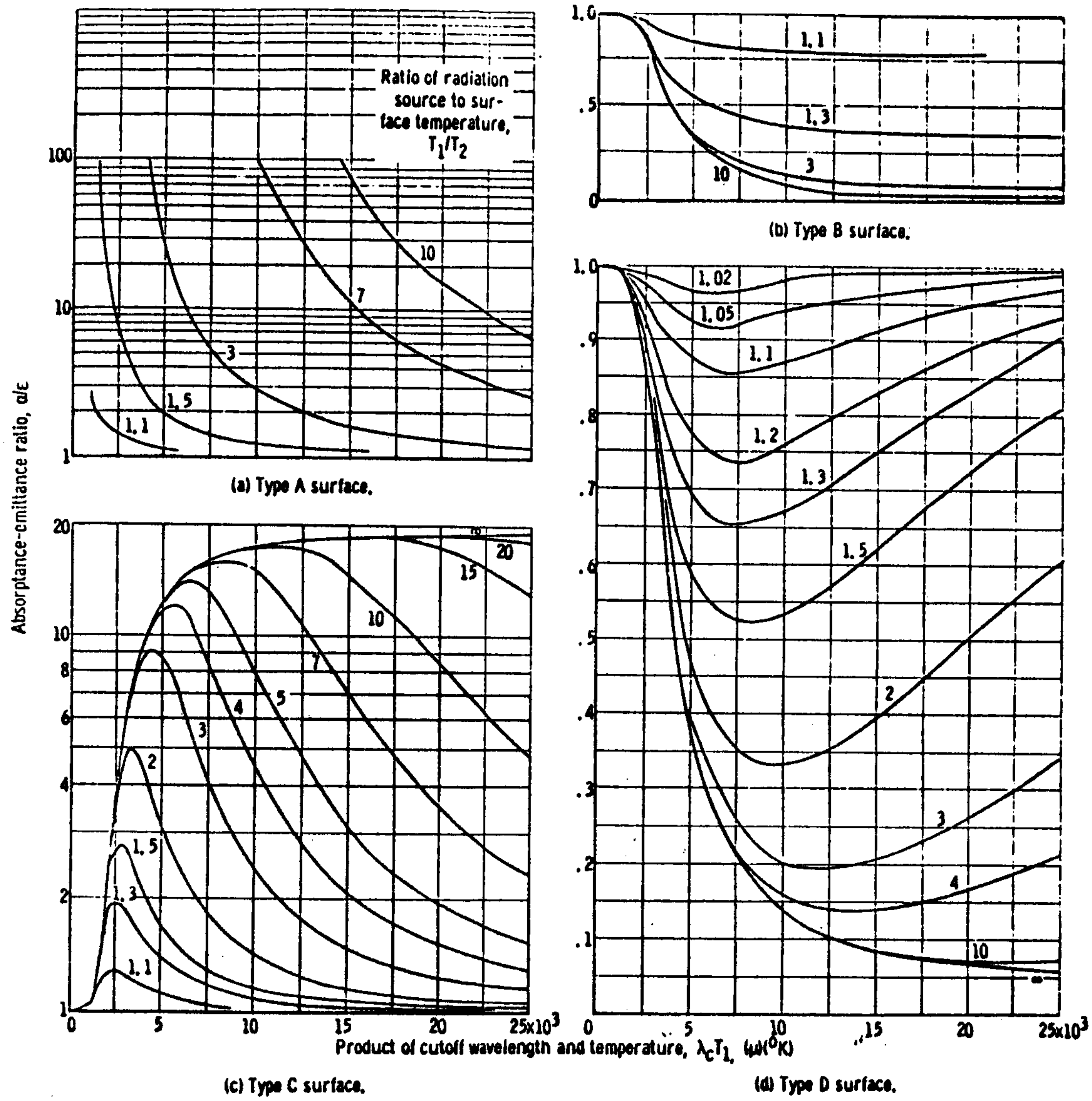

Figure 4. Absorptance-emittance ratio for spectrally selective surfaces as function of wavelength-temperature product and temperature ratio.

cate the present state of the art in this regard.

Highest values of α/ϵ are obtained by coating a good reflector (a polished metal) with a selective absorber for short wavelength radiation, for example, by coating molybdenum with tantalum oxide.

Solar absorptance and emittance data are given for several surface temperatures in reference 4 for tantalum oxide on molybdenum. These are listed in Table I, and α_s/ϵ values are compared with those predicted by the present analysis. The source (Sun) temperature was taken as 6000°K, λ_c as 1.5 microns, and $\lambda_c T_1$ as 9000 micron-°K.

TABLE I. COMPARISON OF MEASURED AND ESTIMATED ABSORPTANCE-EMITTANCE RATIOS

Surface temperature, T_2, °K	Ratio of source to surface temperature, T_1/T_2	Data from ref. 4			α_s/ϵ predicted from Figure 4(c)
		Total absorptance, α_s	Total emittance, ϵ	α_s/ϵ	
298	20	0.83	0.051	16.3	17
533	11.2	.79	.058	13.6	17
819	7.3	.68	.078	8.7	16

It can be seen that the predicted α_s/ϵ closely approaches the experimental value at large temperature ratios. At the smaller temperature ratios (higher surface temperatures) the predicted values are well above those determined experimentally. This difference is largely due to a decrease in α_λ at the shorter wavelengths for this material when its temperature is raised from room temperature. These data are shown in terms of spectral reflectance in Figure 5 which is reproduced from reference 4. The experimental surface is very much different than the type C surface at high temperatures. The surface has α_λ less than 0.95 at short wavelengths and therefore is not a type C surface at higher surface temperatures.

As for surfaces with very low α_s/ϵ values, total absorptance and emittance values for good white paints may achieve values of α_s/ϵ of 0.27 [5]. Recently indications of α_s/ϵ ratios in the order of 0.06 have been obtained with second surface silver coated mirrors. The minimum α_s/ϵ predicted from Figure 4(d) for such a surface is 0.053 with a practical limit of about 0.07. As can be seen, this type experimental surface is predicted from the analysis.

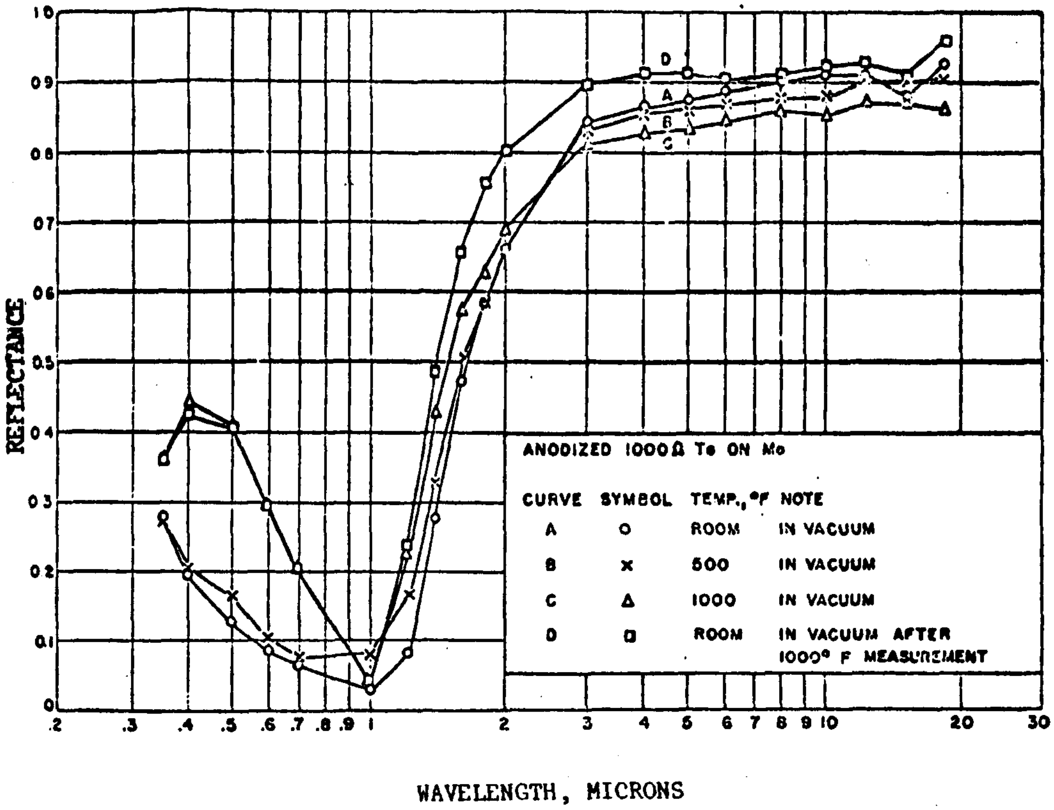

Figure 5. Spectral Reflectance of Tantalum on Molybdenum

To this point the discussion has largely been concerned with the possibility of achieving a high degree of spectral selectivity through having α_λ or ϵ_λ at high values over one wavelength range and at low values over another. The requirements for having a sharp change from one value to another have not been discussed. The importance of this factor depends strongly on the temperature ratio of the system as shown in the following examples.

Consider a system with a temperature ratio T_1/T_2 of 20. At this ratio 98 percent of the energy radiated from a black or gray source at T_1 is at λT less than 17,700 micron-^oK and 98 percent radiated from a similar surface at T_2 is at λT greater than 32,000 micron-^oK. For λT values between 17,700 and 32,000 micron-^oK there is very little energy radiated from either source or receiver. Therefore, the values of α_λ and ϵ_λ that a surface has in this range have very little influence on α, ϵ, or α/ϵ. The surface can have a sharp break at the λ_c value which results in $\lambda_c T$ anywhere between 17,700 and 32,000 micron-^oK, or there can be a slow transition between high and low values of α_λ in this range without significantly changing the performance of the surface.

If the temperature ratio is reduced to 10, 98 percent of the source energy is still at λT less than 17,700 micron-^oK, but 3.5 percent of the energy from the surface is also found below the value. Therefore, at $T_1/T_2 = 10$, there is a small overlap in the spectral distribution from source and surface, and this overlap increases as the temperature ratio decreases. At the lower temperature ratios it becomes increasingly important to have λ_c at the proper wavelength and for α_λ to change as abruptly as possible.

Concluding Remarks

It has been shown that the product of λ_c, a term characterizing the spectral selectivity of a surface, and T_1, the temperature of a black or gray radiation source, can be used to develop a series of completely general curves from which values for the ratio of absorptance to emittance α/ϵ can be read directly. Such curves were developed for surfaces with two degrees of spectral selectivity, those representing a theoretical limit and those that may be within reach experimentally.

The latter curves indicate that the maximum α/ϵ that is likely to be attained is about 18 and the minimum about 0.07. These maximum and minimum values are attainable only at high values of the ratio of radiation source to surface temperature. At lower values of this temperature ratio, α/ϵ will be much closer to unity.

Home Problems

These problems should be solved using Figures 4(c) and 4(d) of the Monograph.

1. What are the maximum and minimum values of α_s/ε that may be achieved on the sunlit surface of a space vehicle at the orbit distance of the Earth from the sun? Assuming the surface is a back insulated plate, what will be the ratio T_1/T_2. Use 6000°K for the temperature of the solar source.

2. What is the maximum temperature which could be obtained by a flat plate solar collector in space at 1AU if the plate faces have an emmitance of 0.05. Assume the solar irradiation at 1AU is 442 Btu/hr-ft^2.

3. What is the minimum temperature of a flat plate radiator, i.e., a radiator which sees the sun on one surface and space with the other surface, under the same conditions as problem 2.

4. What is the minimum temperature for a spherical body at 0.1 AU from the sun? Assume the body is isothermal and that the solar constant is 13.5 watts per square centimeter.

5. What is the maximum temperature which may be achieved under the conditions of problem 4?

6. At what wavelength should the value of α_λ or ε_λ change from a high to a low value to achieve the maximum α_s/ε desired under the conditions of problem 4?

References

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2. Eckert, E. R. G.; and Drake, Robert M: Heat and Mass Transfer. Second ed., McGraw-Hill Book Co., Inc., 1959, p. 371.

3. Liebert, C. H.; and Hibbard, R. R.: Performance of Spectrally Selective Collector Surfaces in a Solar-Driven Carnot Space-Power System. Solar Energy, vol. 6, no. 3, July 1962, pp. 84-88.

4. Schmidt, R. M.; and Janssen, J. E.: Selective Coatings for Vacuum Stable High Temperature Solar Absorbers. Paper Presented at Symposium on Thermal Radiation of Solids, ASD, USAF, NBS, NASA, San Francisco (Calif.), Mar. 4-6, 1964.

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SOLUTION TO HOME PROBLEMS

1. The maximum α_s/ϵ, according to Figure 4(c), will be about 18, and according to Figure 4(d), the minimum α_s/ϵ will be about 0.07. $\frac{T_1}{T_2}$ can be as low as 10 and as high as 20 and the α_s/ϵ values will still hold approximately.

2. For flat plate solar collector, $\alpha_s GA = \epsilon\sigma(A)T_2^4$ **(This assumes that the back side of the collector is insulated such that energy is emitted by the front surface only. If the back side is not insulated, the energy equation would be $\alpha_s GA = \epsilon\sigma(2A)T_2^4$)**

$$T_2^4 = \frac{\alpha_s}{\epsilon}\,\frac{G_1}{\sigma}$$

$$T_2 = 410^{\circ}K\left(\frac{\alpha_s}{\epsilon}\right)^{\frac{1}{4}}$$

Assume $T_2 = 356^{\circ}K$

$$\frac{T_1}{T_2} = 7$$

$$\left(\frac{\alpha_s}{\epsilon}\right)_{max} \cong 16$$

$$T_2 \cong 2(416^{\circ}K)$$

$$T_2 \cong 832^{\circ}K$$

Hence maximum temperature possible is approximately $830^{\circ}K$.

3. For flat plate radiator emitting on both surfaces, but absorbing on one surface,

$$\alpha_s GA = \epsilon\sigma(2A)T_2^4$$

$$T_2 = \left(\frac{\alpha_s}{2\epsilon}\,\frac{G_1}{\sigma}\right)^{\frac{1}{4}}$$

$$\left(\frac{\alpha_s}{\epsilon}\right)_{min} = 0.07$$

$$T_2 = 353^\circ K(.07)^{\frac{1}{4}}$$

$$T_2 = 182^\circ K$$

$$T_1/T_2 = \frac{6000^\circ K}{182^\circ K} = 33$$

$\left(\frac{\alpha_s}{\epsilon}\right)_{min}$ for $T_1/T_2 = 33$ is approximately 0.07

Hence minimum temperature is approximately $\underline{182^\circ K}$.

4. For spherical body:

$$\alpha_s GA = \epsilon\sigma(4A)T_2^4$$

$$T_2 = \left(\frac{\alpha_s G_1}{4\epsilon\sigma}\right)^{\frac{1}{4}}$$

$$T_2 = 878^\circ K\left(\frac{\alpha_s}{\epsilon}\right)^{\frac{1}{4}}$$

Assume $T_2 = 450^\circ K$

$$T_1/T_2 = 13$$

$$\left(\frac{\alpha_s}{\epsilon}\right)_{min} = 0.07$$

$$T_2 = 878^\circ K(.07)^{\frac{1}{4}}$$

$$T_2 = 453^\circ K$$

Hence minimum temperature is approximately $\underline{450^\circ K}$.

5. From Problem 4:

$$T_2 = 878^\circ K\left(\frac{\alpha_s}{\epsilon}\right)^{\frac{1}{4}}$$

Assume $T_2 = 1580^\circ K$

$$T_1/T_2 = 3.8$$

$$\left(\frac{\alpha_s}{\epsilon}\right)_{max} \cong 11$$

$$T_2 = 878^\circ K(11)^{\frac{1}{4}}$$

$T_2 = 1595^\circ K$

Hence maximum temperature is approximately $\underline{1600^\circ K}$.

6. From Figure 4(c), maximum α_s/ϵ for $T_1/T_2 = 3.8$ occurs at approximately $5500\mu^\circ K$. Since $T_1 = 6000^\circ K$

$$\lambda_c = \frac{5500}{6000}$$

$$\lambda_c = 0.917.$$